

Final Exam Review

Simplify. Write "undefined" for expressions that are undefined.

1)
$$\begin{bmatrix} 1 \\ -5 \\ 6 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \\ 6 \end{bmatrix}$$

2)
$$\begin{bmatrix} 4 \\ -5 \\ 3 \\ -4 \end{bmatrix} - \left(\begin{bmatrix} -5 \\ -6 \\ 5 \\ -6 \end{bmatrix} + \begin{bmatrix} -3 \\ -6 \\ -3 \\ 2 \end{bmatrix} \right)$$

3)
$$\begin{bmatrix} -3 & 6 \\ 3 & -3 \end{bmatrix} + \begin{bmatrix} -5 & 1 \\ 3 & -4 \end{bmatrix}$$

4)
$$\begin{bmatrix} -6 & -1 \\ -1 & -2 \\ -2 & 6 \end{bmatrix} + \begin{bmatrix} -1 & 1 & -6 \\ -2 & -1 & 4 \end{bmatrix}$$

5)
$$2 \begin{bmatrix} 6 & -1 \\ -1 & -3 \\ -1 & 5 \end{bmatrix}$$

6)
$$3 \cdot \left(-4 \begin{bmatrix} 3 & -4 \\ -5 & -3 \\ -4 & 3 \end{bmatrix} \right)$$

7)
$$\begin{bmatrix} 5 & -3 \\ -6 & -6 \\ 4 & -6 \end{bmatrix} \cdot \begin{bmatrix} -6 & -5 \\ -5 & -1 \end{bmatrix}$$

8)
$$\begin{bmatrix} 1 & -2 & 0 & -5 \\ 5 & -6 & -6 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & -3 \\ -4 & 5 \end{bmatrix}$$

9)
$$\begin{bmatrix} 4 & -4 \\ -2 & -3 \\ 3 & -6 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ -3 & 1 \end{bmatrix}$$

10)
$$\begin{bmatrix} 4 & -4 \\ -1 & 0 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} -4 & 0 \\ -2 & 1 \end{bmatrix}$$

Evaluate each determinant.

$$11) \begin{vmatrix} -5 & -4 \\ -1 & -3 \end{vmatrix}$$

$$12) \begin{vmatrix} 0 & -3 \\ -2 & 0 \end{vmatrix}$$

$$13) \begin{vmatrix} -3 & -3 & 0 \\ -4 & -5 & 0 \\ -1 & 4 & -5 \end{vmatrix}$$

$$14) \begin{vmatrix} -1 & 5 & -3 \\ 0 & -1 & -4 \\ 5 & -1 & -3 \end{vmatrix}$$

Find the inverse of each matrix.

$$15) \begin{bmatrix} 0 & -7 \\ 0 & 3 \end{bmatrix}$$

$$16) \begin{bmatrix} -5 & 8 \\ 0 & 6 \end{bmatrix}$$

$$17) \begin{bmatrix} 2 & -1 \\ -10 & 10 \end{bmatrix}$$

$$18) \begin{bmatrix} 4 & 1 \\ -4 & 1 \end{bmatrix}$$

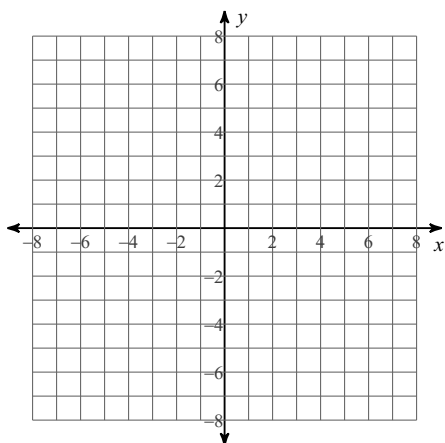
Identify the vertices, co-vertices, and foci of each.

$$19) \frac{(x+7)^2}{36} + \frac{y^2}{100} = 1$$

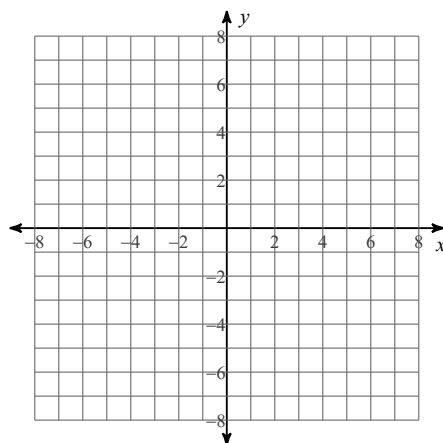
$$20) \frac{(x+10)^2}{196} + \frac{(y+6)^2}{64} = 1$$

Identify the vertices, co-vertices, and foci of each. Then sketch the graph.

$$21) \frac{(x+1)^2}{36} + \frac{(y+1)^2}{25} = 1$$



$$22) \frac{x^2}{16} + (y+2)^2 = 1$$



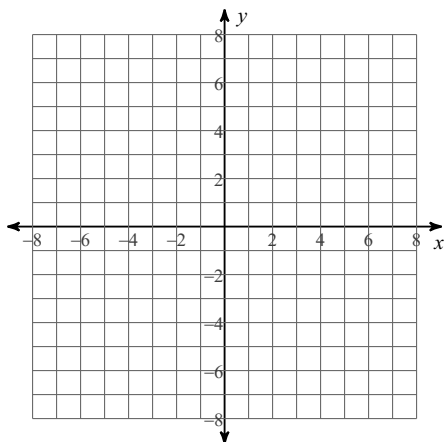
Use the information provided to write the standard form equation of each ellipse.

$$23) x^2 + 49y^2 - 14x - 196y + 49 = 0$$

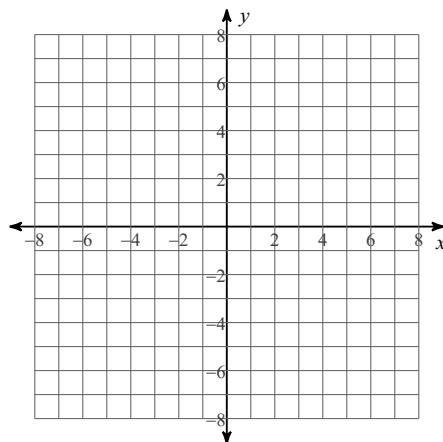
$$24) 9x^2 + 4y^2 - 36x + 24y - 252 = 0$$

Identify the vertices, foci, direction of opening, length of the transverse axis, and length of the conjugate axis of each. Then sketch the graph.

$$25) \frac{(x+1)^2}{9} - \frac{y^2}{25} = 1$$



$$26) (y+4)^2 - \frac{x^2}{25} = 1$$



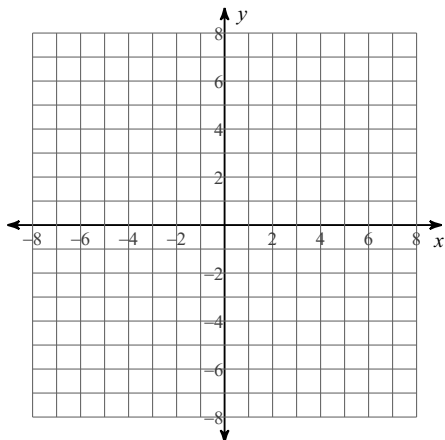
Use the information provided to write the standard form equation of each hyperbola.

$$27) x^2 - 4y^2 - 16x - 32y - 64 = 0$$

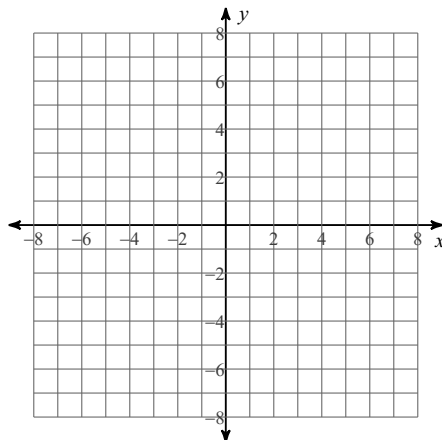
$$28) -x^2 + 4y^2 - 2x + 48y + 43 = 0$$

Identify the vertex, focus, directrix, direction of opening, and length of the latus rectum of each. Then sketch the graph.

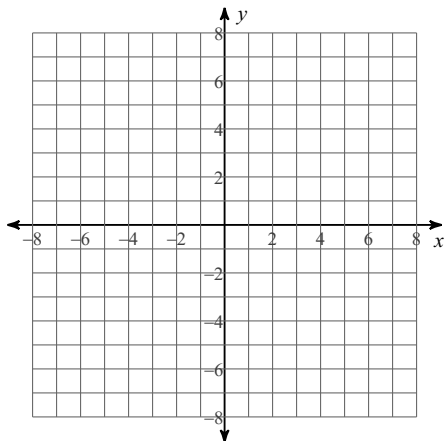
29) $2y = (x - 1)^2$



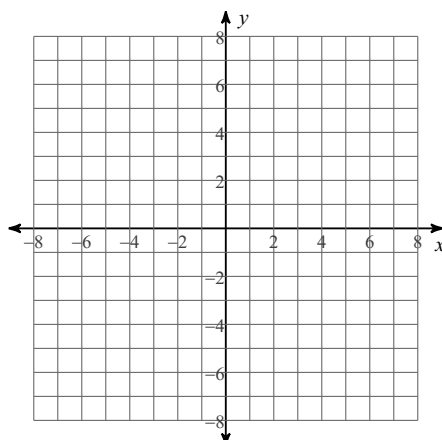
30) $-3(x + 3) = (y - 3)^2$



31) $x - 3 = (y - 1)^2$



32) $-\frac{1}{2}(y + 4) = (x - 3)^2$



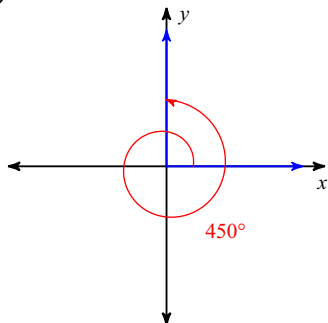
Use the information provided to write the vertex form equation of each parabola.

33) Vertex: $(4, -3)$, Directrix: $y = -\frac{5}{2}$

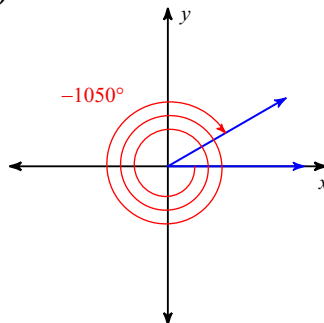
34) Vertex: $(-5, -7)$, Focus: $(-5, -6)$

Find the exact value of each trigonometric function.

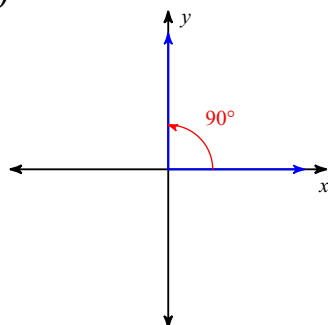
35) $\tan \theta$



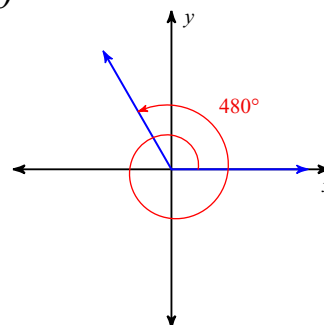
36) $\sin \theta$



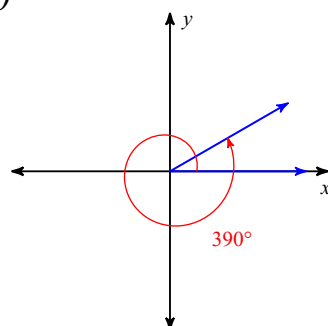
37) $\sec \theta$



38) $\csc \theta$

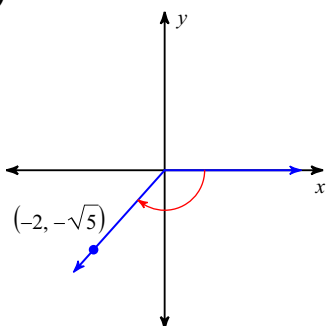


39) $\cos \theta$

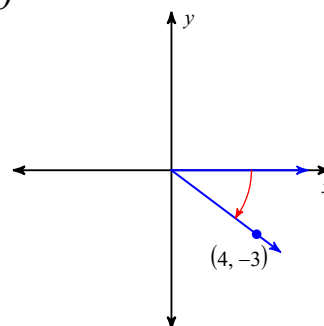


Use the given point on the terminal side of angle θ to find the value of the trigonometric function indicated.

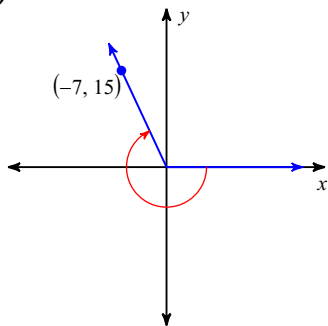
40) $\sin \theta$



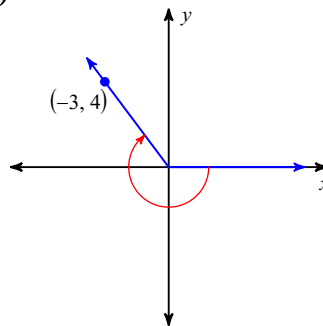
41) $\sec \theta$



42) $\tan \theta$



43) $\tan \theta$



Find the exact values of the five trigonometric ratios not given.

44) $\cos \theta = -\frac{4}{5}$ and $\sin \theta < 0$

45) $\sin \theta = \frac{3}{5}$ and $\cos \theta > 0$

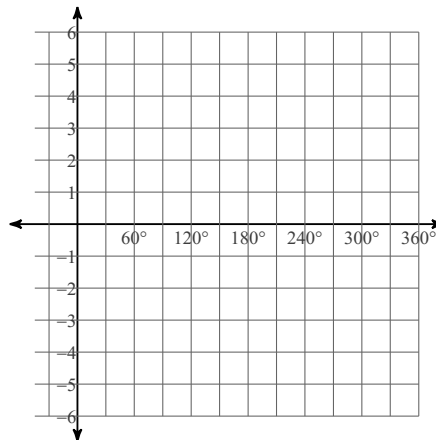
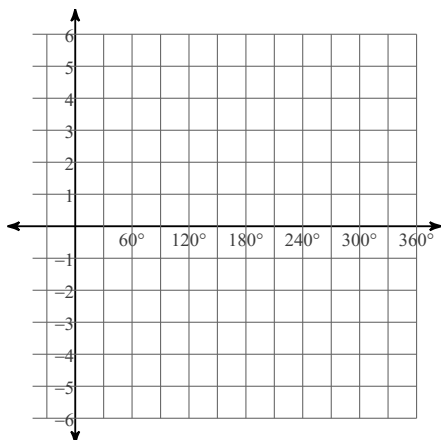
46) $\sec \theta = -\frac{\sqrt{13}}{3}$ and $\sin \theta < 0$

47) $\sec \theta = \frac{\sqrt{5}}{2}$ and $\sin \theta < 0$

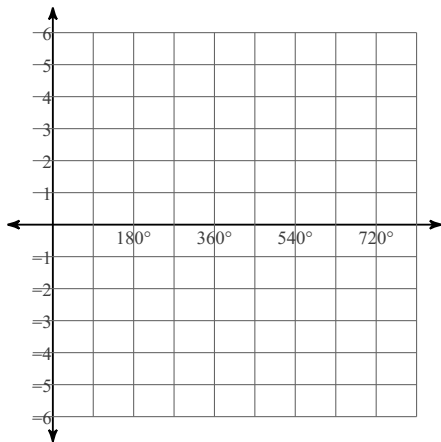
Find the amplitude, the period in degrees, the phase shift in degrees, and the vertical shift. Then sketch the graph using degrees.

48) $y = 4\sin(4\theta + 30) - 1$

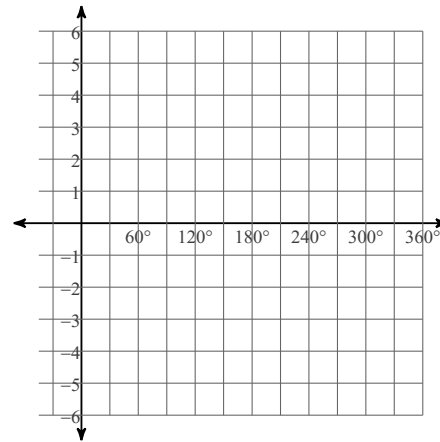
49) $y = -1 + 2\cot(2\theta + 120)$



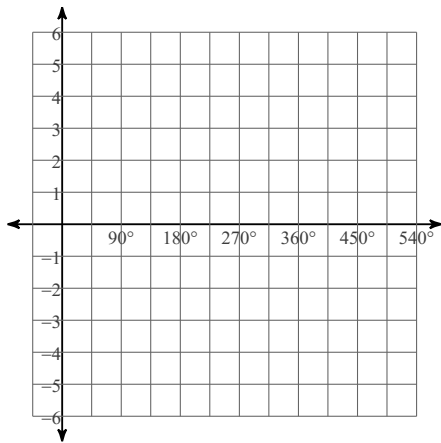
$$50) y = 4 \tan \left(\frac{\theta}{3} + 90 \right)$$



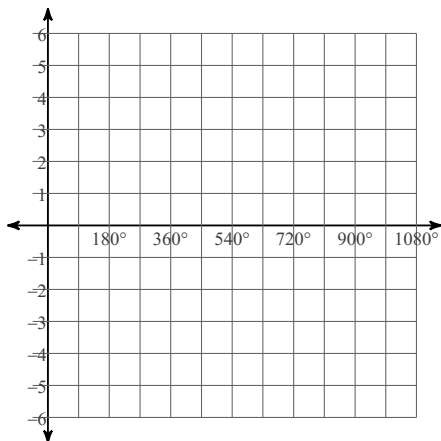
$$51) y = 3 \cos (2\theta + 150) - 1$$



$$52) y = 3 \csc (\theta - 30)$$



$$53) y = 2 + 2 \sec \left(\frac{\theta}{2} + 210 \right)$$



Find the exact value of each expression.

54) $\cos^{-1} 1$

55) $\sin^{-1} -1$

56) $\cos^{-1} -\frac{\sqrt{3}}{2}$

57) $\sin^{-1} \left(\cot \frac{\pi}{2} \right)$

58) $\cos^{-1} \left(\cot \frac{3\pi}{4} \right)$

59) $\tan^{-1} \left(\sin -\frac{\pi}{2} \right)$

Final Exam Review

Simplify. Write "undefined" for expressions that are undefined.

$$1) \begin{bmatrix} 1 \\ -5 \\ 6 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ -3 \\ 12 \end{bmatrix}$$

$$2) \begin{bmatrix} 4 \\ -5 \\ 3 \\ -4 \end{bmatrix} - \left(\begin{bmatrix} -5 \\ -6 \\ 5 \\ -6 \end{bmatrix} + \begin{bmatrix} -3 \\ -6 \\ -3 \\ 2 \end{bmatrix} \right)$$

$$\begin{bmatrix} 12 \\ 7 \\ 1 \\ 0 \end{bmatrix}$$

$$3) \begin{bmatrix} -3 & 6 \\ 3 & -3 \end{bmatrix} + \begin{bmatrix} -5 & 1 \\ 3 & -4 \end{bmatrix}$$

$$\begin{bmatrix} -8 & 7 \\ 6 & -7 \end{bmatrix}$$

$$4) \begin{bmatrix} -6 & -1 \\ -1 & -2 \\ -2 & 6 \end{bmatrix} + \begin{bmatrix} -1 & 1 & -6 \\ -2 & -1 & 4 \end{bmatrix}$$

Undefined

$$5) 2 \begin{bmatrix} 6 & -1 \\ -1 & -3 \\ -1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 12 & -2 \\ -2 & -6 \\ -2 & 10 \end{bmatrix}$$

$$6) 3 \cdot \left(\begin{bmatrix} 3 & -4 \\ -4 & -5 \\ -4 & 3 \end{bmatrix} \right)$$

$$\begin{bmatrix} -36 & 48 \\ 60 & 36 \\ 48 & -36 \end{bmatrix}$$

$$7) \begin{bmatrix} 5 & -3 \\ -6 & -6 \\ 4 & -6 \end{bmatrix} \cdot \begin{bmatrix} -6 & -5 \\ -5 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -15 & -22 \\ 66 & 36 \\ 6 & -14 \end{bmatrix}$$

$$8) \begin{bmatrix} 1 & -2 & 0 & -5 \\ 5 & -6 & -6 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & -3 \\ -4 & 5 \end{bmatrix}$$

Undefined

$$9) \begin{bmatrix} 4 & -4 \\ -2 & -3 \\ 3 & -6 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ -3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 0 \\ 9 & -5 \\ 18 & -3 \end{bmatrix}$$

$$10) \begin{bmatrix} 4 & -4 \\ -1 & 0 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} -4 & 0 \\ -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -8 & -4 \\ 4 & 0 \\ -20 & 4 \end{bmatrix}$$

Evaluate each determinant.

$$11) \begin{vmatrix} -5 & -4 \\ -1 & -3 \end{vmatrix}$$

11

$$12) \begin{vmatrix} 0 & -3 \\ -2 & 0 \end{vmatrix}$$

-6

$$13) \begin{vmatrix} -3 & -3 & 0 \\ -4 & -5 & 0 \\ -1 & 4 & -5 \end{vmatrix}$$

-15

$$14) \begin{vmatrix} -1 & 5 & -3 \\ 0 & -1 & -4 \\ 5 & -1 & -3 \end{vmatrix}$$

-114

Find the inverse of each matrix.

$$15) \begin{bmatrix} 0 & -7 \\ 0 & 3 \end{bmatrix}$$

No inverse exists

$$16) \begin{bmatrix} -5 & 8 \\ 0 & 6 \end{bmatrix}$$

$$-\frac{1}{30} \cdot \begin{bmatrix} 6 & -8 \\ 0 & -5 \end{bmatrix}$$

$$17) \begin{bmatrix} 2 & -1 \\ -10 & 10 \end{bmatrix}$$

$$\frac{1}{10} \cdot \begin{bmatrix} 10 & 1 \\ 10 & 2 \end{bmatrix}$$

$$18) \begin{bmatrix} 4 & 1 \\ -4 & 1 \end{bmatrix}$$

$$\frac{1}{8} \cdot \begin{bmatrix} 1 & -1 \\ 4 & 4 \end{bmatrix}$$

Identify the vertices, co-vertices, and foci of each.

$$19) \frac{(x+7)^2}{36} + \frac{y^2}{100} = 1$$

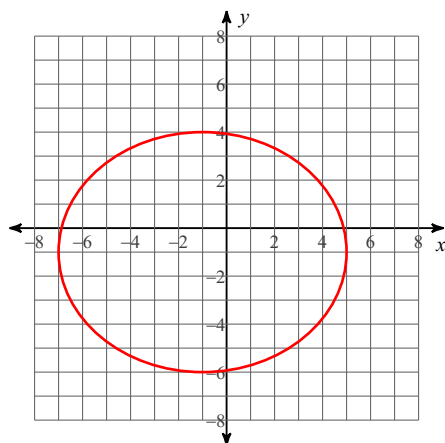
Vertices: $(-7, 10), (-7, -10)$
Co-vertices: $(-1, 0), (-13, 0)$
Foci: $(-7, 8), (-7, -8)$

$$20) \frac{(x+10)^2}{196} + \frac{(y+6)^2}{64} = 1$$

Vertices: $(4, -6), (-24, -6)$
Co-vertices: $(-10, 2), (-10, -14)$
Foci: $(-10 + 2\sqrt{33}, -6), (-10 - 2\sqrt{33}, -6)$

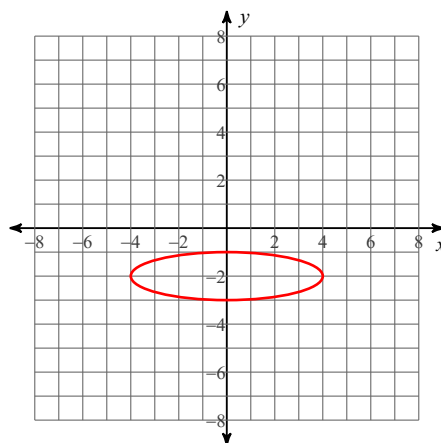
Identify the vertices, co-vertices, and foci of each. Then sketch the graph.

$$21) \frac{(x+1)^2}{36} + \frac{(y+1)^2}{25} = 1$$



Vertices: $(5, -1)$
 $(-7, -1)$
 Co-vertices: $(-1, 4)$
 $(-1, -6)$
 Foci: $(-1 + \sqrt{11}, -1)$
 $(-1 - \sqrt{11}, -1)$

$$22) \frac{x^2}{16} + (y+2)^2 = 1$$



Vertices: $(4, -2)$
 $(-4, -2)$
 Co-vertices: $(0, -1)$
 $(0, -3)$
 Foci: $(\sqrt{15}, -2)$
 $(-\sqrt{15}, -2)$

Use the information provided to write the standard form equation of each ellipse.

$$23) x^2 + 49y^2 - 14x - 196y + 49 = 0$$

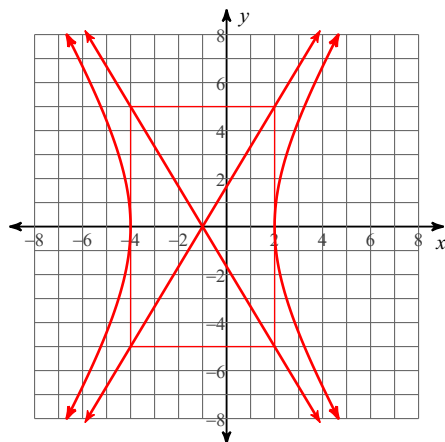
$$\frac{(x-7)^2}{196} + \frac{(y-2)^2}{4} = 1$$

$$24) 9x^2 + 4y^2 - 36x + 24y - 252 = 0$$

$$\frac{(x-2)^2}{36} + \frac{(y+3)^2}{81} = 1$$

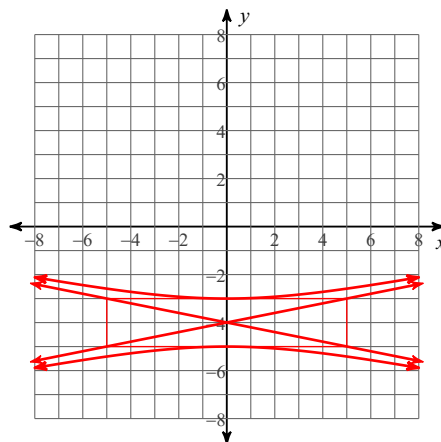
Identify the vertices, foci, direction of opening, length of the transverse axis, and length of the conjugate axis of each. Then sketch the graph.

$$25) \frac{(x+1)^2}{9} - \frac{y^2}{25} = 1$$



Vertices: $(2, 0)$
 $(-4, 0)$
 Foci: $(-1 + \sqrt{34}, 0)$
 $(-1 - \sqrt{34}, 0)$
 Opens left/right
 Transverse Axis: 6 units
 Conjugate Axis: 10 units

$$26) (y+4)^2 - \frac{x^2}{25} = 1$$



Vertices: $(0, -3)$
 $(0, -5)$
 Foci: $(0, -4 + \sqrt{26})$
 $(0, -4 - \sqrt{26})$
 Opens up/down
 Transverse Axis: 2 units
 Conjugate Axis: 10 units

Use the information provided to write the standard form equation of each hyperbola.

$$27) x^2 - 4y^2 - 16x - 32y - 64 = 0$$

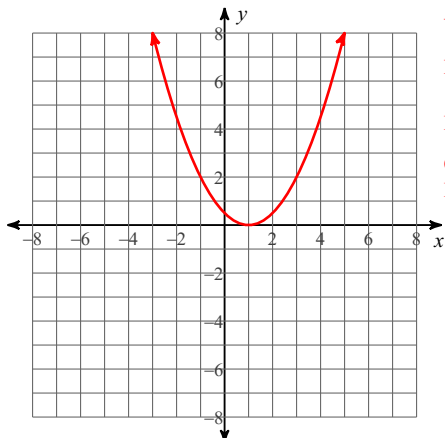
$$\frac{(x-8)^2}{64} - \frac{(y+4)^2}{16} = 1$$

$$28) -x^2 + 4y^2 - 2x + 48y + 43 = 0$$

$$\frac{(y+6)^2}{25} - \frac{(x+1)^2}{100} = 1$$

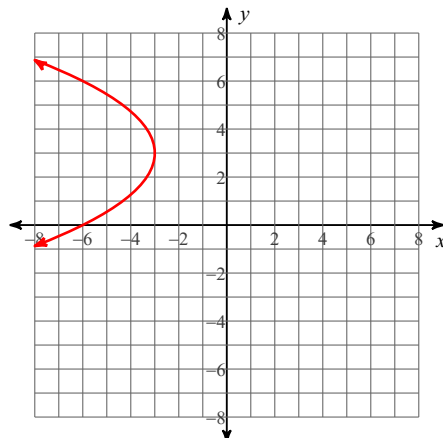
Identify the vertex, focus, directrix, direction of opening, and length of the latus rectum of each. Then sketch the graph.

29) $2y = (x - 1)^2$



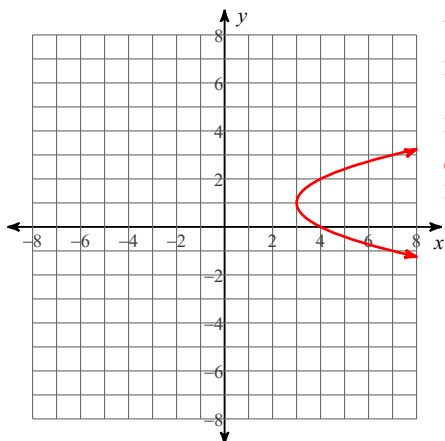
Vertex: $(1, 0)$
 Focus: $(1, \frac{1}{2})$
 Directrix: $y = -\frac{1}{2}$
 Opens: Up
 Latus rectum: 2 units

30) $-3(x + 3) = (y - 3)^2$



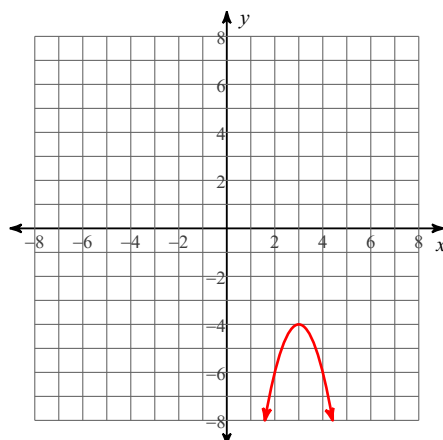
Vertex: $(-3, 3)$
 Focus: $(-\frac{15}{4}, 3)$
 Directrix: $x = -\frac{9}{4}$
 Opens: Left
 Latus rectum: 3 units

31) $x - 3 = (y - 1)^2$



Vertex: $(3, 1)$
 Focus: $(\frac{13}{4}, 1)$
 Directrix: $x = \frac{11}{4}$
 Opens: Right
 Latus rectum: 1 unit

32) $-\frac{1}{2}(y + 4) = (x - 3)^2$



Vertex: $(3, -4)$
 Focus: $(3, -\frac{33}{8})$
 Directrix: $y = -\frac{31}{8}$
 Opens: Down
 Latus rectum: $\frac{1}{2}$ units

Use the information provided to write the vertex form equation of each parabola.

33) Vertex: $(4, -3)$, Directrix: $y = -\frac{5}{2}$

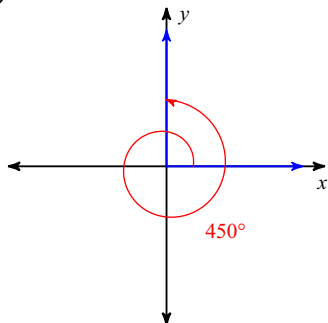
$$y = -\frac{1}{2}(x - 4)^2 - 3$$

34) Vertex: $(-5, -7)$, Focus: $(-5, -6)$

$$y = \frac{1}{4}(x + 5)^2 - 7$$

Find the exact value of each trigonometric function.

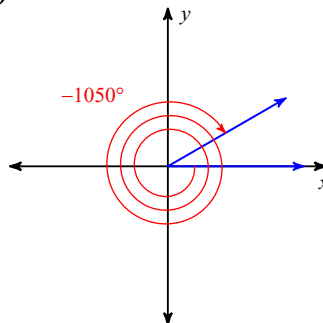
35) $\tan \theta$



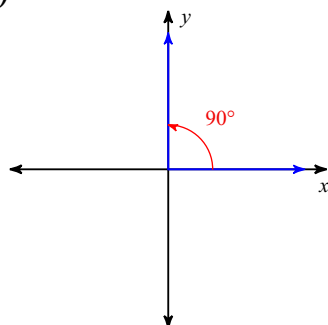
Undefined

36) $\sin \theta$

$\frac{1}{2}$



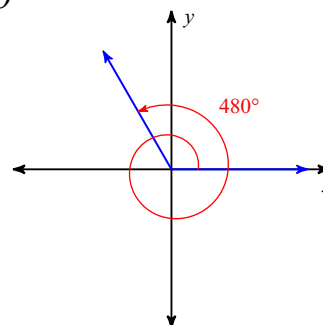
37) $\sec \theta$



Undefined

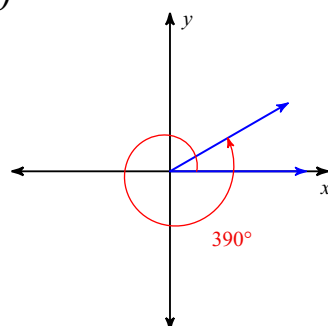
38) $\csc \theta$

$\frac{2\sqrt{3}}{3}$



39) $\cos \theta$

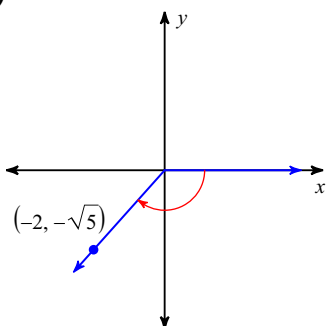
$\frac{\sqrt{3}}{2}$



Use the given point on the terminal side of angle θ to find the value of the trigonometric function indicated.

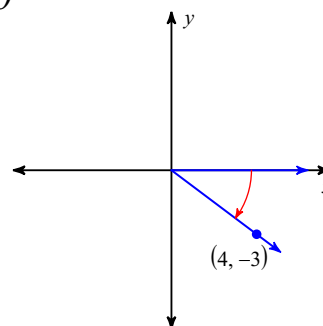
40) $\sin \theta$

$-\frac{\sqrt{5}}{3}$

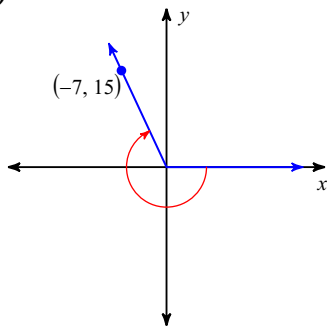


41) $\sec \theta$

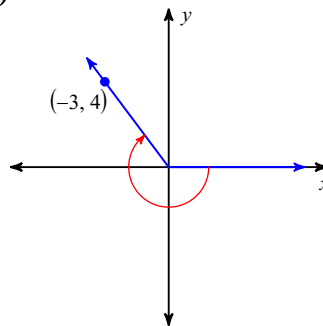
$\frac{5}{4}$



42) $\tan \theta = -\frac{15}{7}$



43) $\tan \theta = -\frac{4}{3}$



Find the exact values of the five trigonometric ratios not given.

44) $\cos \theta = -\frac{4}{5}$ and $\sin \theta < 0$

$\sin \theta = -\frac{3}{5}, \tan \theta = \frac{3}{4}$

$\csc \theta = -\frac{5}{3}, \sec \theta = -\frac{5}{4}, \cot \theta = \frac{4}{3}$

45) $\sin \theta = \frac{3}{5}$ and $\cos \theta > 0$

$\cos \theta = \frac{4}{5}, \tan \theta = \frac{3}{4}$

$\csc \theta = \frac{5}{3}, \sec \theta = \frac{5}{4}, \cot \theta = \frac{4}{3}$

46) $\sec \theta = -\frac{\sqrt{13}}{3}$ and $\sin \theta < 0$

$\sin \theta = -\frac{2\sqrt{13}}{13}, \cos \theta = -\frac{3\sqrt{13}}{13}, \tan \theta = \frac{2}{3}$

$\csc \theta = -\frac{\sqrt{13}}{2}, \cot \theta = \frac{3}{2}$

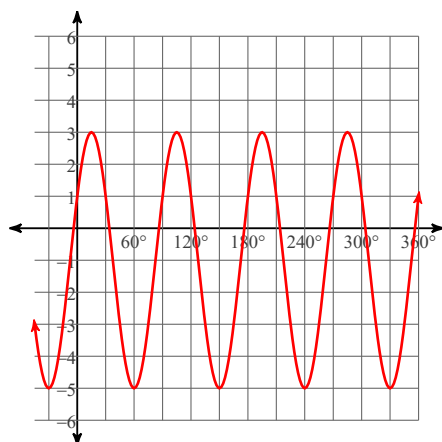
47) $\sec \theta = \frac{\sqrt{5}}{2}$ and $\sin \theta < 0$

$\sin \theta = -\frac{\sqrt{5}}{5}, \cos \theta = \frac{2\sqrt{5}}{5}, \tan \theta = -\frac{1}{2}$

$\csc \theta = -\sqrt{5}, \cot \theta = -2$

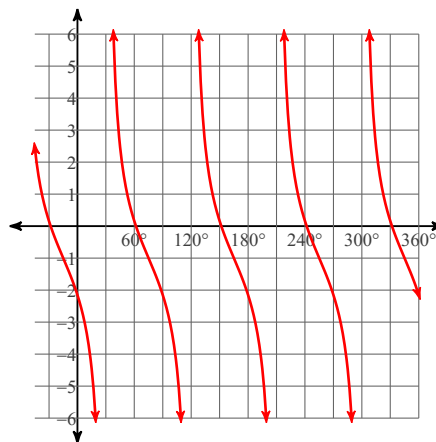
Find the amplitude, the period in degrees, the phase shift in degrees, and the vertical shift. Then sketch the graph using degrees.

48) $y = 4\sin(4\theta + 30) - 1$



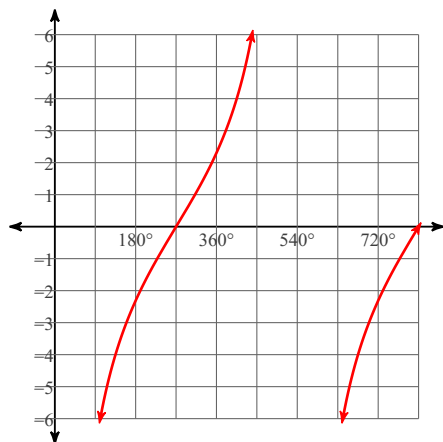
Amplitude: 4
Period: 90°
Phase shift: Left 7.5°
Vert. shift: Down 1

49) $y = -1 + 2\cot(2\theta + 120)$



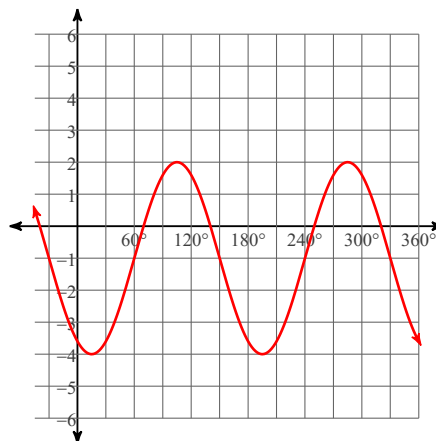
Amplitude: None
Period: 90°
Phase shift: Left 60°
Vert. shift: Down 1

$$50) y = 4\tan\left(\frac{\theta}{3} + 90\right)$$



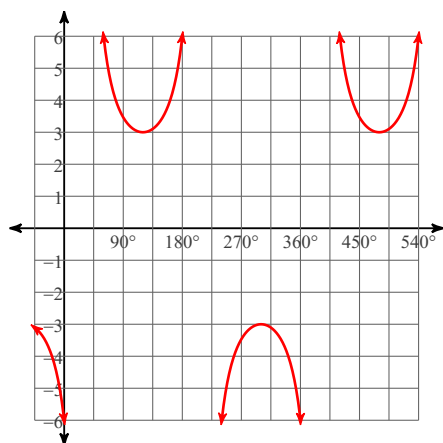
Amplitude: None
 Period: 540°
 Phase shift: Left 270°
 Vert. shift: None

$$51) y = 3\cos(2\theta + 150) - 1$$



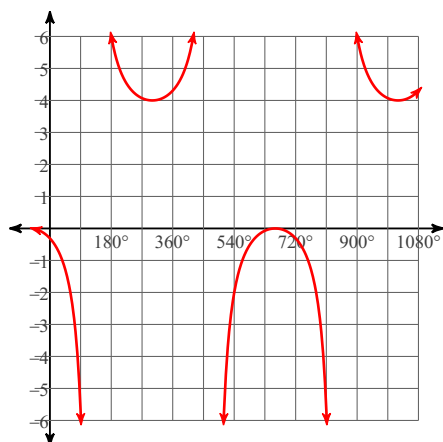
Amplitude: 3
 Period: 180°
 Phase shift: Left 75°
 Vert. shift: Down 1

$$52) y = 3\csc(\theta - 30)$$



Amplitude: None
 Period: 360°
 Phase shift: Right 30°
 Vert. shift: None

$$53) y = 2 + 2\sec\left(\frac{\theta}{2} + 210\right)$$



Amplitude: None
 Period: 720°
 Phase shift: Left 420°
 Vert. shift: Up 2

Find the exact value of each expression.

54) $\cos^{-1} 1$

0

55) $\sin^{-1} -1$

$-\frac{\pi}{2}$

56) $\cos^{-1} -\frac{\sqrt{3}}{2}$

$\frac{5\pi}{6}$

57) $\sin^{-1} \left(\cot \frac{\pi}{2} \right)$

0

58) $\cos^{-1} \left(\cot \frac{3\pi}{4} \right)$

π

59) $\tan^{-1} \left(\sin -\frac{\pi}{2} \right)$

$-\frac{\pi}{4}$